

at the Distance of above 120 Feet, and Silver at 50. The Theory which led me to this Discovery is founded upon two important Remarks, the one that the Heat is not proportional to the Quantity of Light, and the other that the Rays do not come parallel from the Sun. The first of those, which appears to be a Paradox, is nevertheless a Truth, of which one may easily satisfy one's self, by reflecting that Heat propagates itself even within Bodies; and that when one heats at the same time a large Superficies, the Firing is much quicker than when one only heats a small Portion of the same.

I am, &c.

*From the Chateau de Montbard in  
Burgundy, Sept. 18. N.S. 1748.*

Buffon.

V. *An Essay on Quantity; occasioned by reading a Treatise, in which Simple and Compound Ratio's are applied to Virtue and Merit, by the Rev. Mr. Reid; communicated in a Letter from the Rev. Henry Miles D.D. & F.R.S. to Martin Folkes Esq; Pr.R.S.*

SECT. I.

*What Quantity is.*

Read Nov. 3. 1748 SINCE mathematical Demonstration is thought to carry a peculiar Evidence along with it, which leaves no Room for further Dispute; it may be of some Use, or Entertainment

tainment at least, to inquire to what Subjects this kind of Proof may be applied.

Mathematics contain properly the Doctrine of Measure; and the Object of this Science is commonly said to be Quantity; therefore Quantity ought to be defined, What may be measured. Those who have defined Quantity to be whatever is capable of More or Less, have given too wide a Notion of it, which I apprehend has led some Persons to apply mathematical Reasoning to Subjects that do not admit of it.

Pain and Pleasure admit of various Degrees, but who can pretend to measure them? Had this been possible, it is not to be doubted but we should have had as distinct Names for their various Degrees, as we have for Measures of Length or Capacity; and a Patient should have been able to describe the Quantity of his Pain, as well as the Time it began, or the Part it affected. To talk intelligibly of the Quantity of Pain, we should have some Standard to measure it by; some known Degree of it so well ascertained, that all Men, when they talked of it, should mean the same thing; we should also be able to compare other Degrees of Pain with this, so as to perceive distinctly, not only whether they exceed or fall short of it, but how far, or in what proportion; whether by an half, a fifth, or a tenth.

Whatever has Quantity, or is measurable, must be made up of Parts, which bear Proportion to one another, and to the Whole; so that it may be increased by Addition of like Parts, and diminished by Subtraction, may be multiplied and divided, and in a Word, may bear any Proportion to another

Quantity of the same kind, that one Line or Number can bear to another. That this is essential to all mathematical Quantity, is evident from the first Elements of Algebra, which treats of Quantity in general, or of those Relations and Properties which are common to all Kinds of Quantity. Every algebraical Quantity is supposed capable not only of being increased and diminished, but of being exactly doubled, tripled, halved, or of bearing any assignable Proportion to another Quantity of the same kind. This then is the Characteristic of Quantity; whatever has this Property may be adopted into Mathematics; and its Quantity and Relations may be measured with mathematical Accuracy and Certainty.

## SECT. 2.

### *Of Proper and Improper Quantity.*

There are some Quantities which may be called *Proper*, and others *Improper*. This Distinction is taken notice of by *Aristotle*; but it deserves some Explication.

I call that *Proper* Quantity which is measured by its own Kind; or which of its own Nature is capable of being doubled or tripled, without taking in any Quantity of a different Kind as a Measure of it. Thus a Line is measured by known Lines, as Inches, Feet, or Miles; and the Length of a Foot being known, there can be no Question about the Length of two Feet, or of any Part or Multiple of a Foot. And this known Length, by being multiplied or divided, is sufficient to give us a distinct Idea of any Length whatsoever.

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*Improper*

*Improper* Quantity is that which cannot be measured by its own Kind; but to which we assign a Measure by the means of some proper Quantity that is related to it. Thus Velocity of Motion, when we consider it by itself, cannot be measured. We may perceive one Body to move faster, another slower; but we can have no distinct Idea of a Proportion or *Ratio* between their Velocities, without taking in some Quantity of another Kind to measure them by. Having therefore observed, that by a greater Velocity a greater Space is passed over in the same time, by a less Velocity a less Space, and by an equal Velocity an equal Space; we hence learn to measure Velocity by the Space passed over in a given Time, and to reckon it to be in exact Proportion to that Space: And having once assigned this Measure to it, we can then, and not till then, conceive one Velocity to be exactly double, or half, or in any other Proportion to another; we may then introduce it into mathematical Reasoning without Danger of Confusion, or Error, and may also use it as a Measure of other Improper Quantities.

All the Kinds of Proper Quantity we know, may, I think, be reduced to these four, Extension, Duration, Number, and Proportion. Tho' Proportion be measurable in its own Nature, and therefore hath Proper Quantity, yet as Things cannot have Proportion which have not Quantity of some other Kind, it follows, that whatever has Quantity must have it in one or other of these three Kinds, Extension, Duration, or Number. These are the Measures of themselves, and of all Things else that are measurable.

Number

Number is applicable to somethings, to which it is not commonly applied by the Vulgar. Thus, by attentive Consideration, Lots and Chances of various Kinds appear to be made up of a determinate Number of Chances that are allowed to be equal; and by numbering these, the Values and Proportions of those which are compounded of them may be demonstrated.

Velocity, the Quantity of Motion, Density, Elasticity, the *Vis insita*, and *impressa*, the various Kinds of centripetal Forces, and different Orders of Fluxions, are all Improper Quantities; which therefore ought not to be admitted into Mathematics, without having a Measure of them assigned. The Measure of an improper Quantity ought always to be included in the Definition of it; for it is the giving it a Measure that makes it a proper Subject of mathematical Reasoning. If all Mathematicians had considered, this as carefully as Sir *Isaac Newton* appears to have done, some Labour had been saved both to themselves and to their Readers. That Great Man, whose clear and comprehensive Understanding appears, even in his Definitions, having frequent Occasion to treat of such improper Quantities, never fails to define them, so as to give a Measure of them, either in proper Quantities, or in such as had a known Measure. This may be seen in the Definitions prefixed to his *Princip. Phil. Nat. Math.*

It is not easy to say how many Kinds of improper Quantity, may in time, be introduced into Mathematics, or to what new Subjects Measures may be applied: But this I think we may conclude,

that there is no Foundation in Nature for, nor can any valuable End be served by applying Measure to any, thing but what has these two Properties. First it must admit of Degrees of greater and less. Secondly, it must be associated with, or related to something that has proper Quantity, so as that when one is increased the other is increased, when one is diminished, the other is diminished also; and every Degree of the one must have a determinate Magnitude or Quantity of the other corresponding to it.

It sometimes happens, that we have Occasion to apply different Measures to the same thing. Centripetal Force, as defined by *Newton*, may be measured various Ways, he himself gives different Measures of it, and distinguishes them by different Names, as may be seen in the above-mentioned Definitions.

In reality, I conceive that the applying of Measures to things that properly have not Quantity, is only a Fiction or Artifice of the Mind, for enabling us to conceive more easily, and more distinctly to express and demonstrate, the Properties and Relations of those things that have real Quantity. The Propositions contained in the two first Books of *Newton's Principia* might perhaps be expressed and demonstrated, without those various Measures of Motion, and of centripetal and impressed Forces which he uses: But this would occasion such intricate and perplexed Circumlocutions, and such a tedious Length of Demonstrations as would fright any sober Person from attempting to read them.

SECT. 3. *Coroll. first.*

From the Nature of Quantity we may see what it is that gives Mathematics such Advantage over other Sciences, in Clearness and Certainty; namely, that Quantity admits of a much greater Variety of Relations than any other Subject of human Reasoning; and at the same time every Relation or Proportion of Quantities may by the Help of Lines and Numbers be so distinctly defined, as to be easily distinguished from all others, without any Danger of Mistake. Hence it is that we are able to trace its Relations through a long Process of Reasoning, and with a Perspicuity and Accuracy which we in vain expect in Subjects not capable of Mensuration.

Extended Quantities, such as Lines, Surfaces and Solids, besides what they have in common with all other Quantities, have this peculiar, That their Parts have a particular Place and Disposition among themselves: A Line may not only bear any assignable Proportion to another, in Length or Magnitude, but Lines of the same Length may vary in the Disposition of their Parts; one may be streight, another may be Part of a Curve of any Kind or Dimension, of which there is an endless Variety. The like may be said of Surfaces and Solids. So that extended Quantities admit of no less Variety with regard to their Form than with regard to their Magnitude: And as their various Forms may be exactly defined and measured, no less than their Magnitudes, hence it is that Geometry, which treats of extended Quantity, leads us into a much greater Compass and Variety of Reasoning than any other Branch  
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of Mathematics. Long Deductions in Algebra for the most part are made, not so much by a Train of Reasoning in the Mind, as by an artificial kind of Operation, which is built on a few very simple Principles: But in Geometry we may build one Proposition upon another, a third upon that, and so on, without ever coming to a Limit which we cannot exceed. The Properties of the more simple Figures can hardly be exhausted, much less those of the more complex ones.

#### SECT. 4. *Coroll. 2.*

It may I think be deduced from what hath been above said, That mathematical Evidence is an Evidence *sui generis*, not competent to any Proposition which does not express a Relation of Things measurable by Lines or Numbers. All proper Quantity may be measured by these, and improper Quantities must be measured by those that are proper.

There are many Things capable of More and Less, which perhaps are not capable of Mensuration. Tastes, Smells, the Sensations of Heat and Cold, Beauty, Pleasure, all the Affections and Appetites of the Mind, Wisdom, Folly, and most Kinds of Probability, with many other Things too tedious to enumerate, admit of Degrees, but have not yet been reduced to Measure, nor, as I apprehend, ever can be. I say, most Kinds of Probability, because one Kind of it, *viz.* the Probability of Chances is properly measurable by Number, as is above observed.

Altho' Attempts have been made to apply mathematical Reasoning to some of these Things, and the Quantity of Virtue and Merit in Actions has been  
measured

measured by simple and compound *Ratio's*; yet I do not think that any real Knowledge has been struck out this Way: It may perhaps, if discretely used, be a Help to Discourse on these Subjects, by pleasing the Imagination, and illustrating what is already known; but until our Affections and Appetites shall themselves be reduced to Quantity, and exact Measures of their various Degrees be assigned, in vain shall we essay to measure Virtue and Merit by them. This is only to ring Changes upon Words, and to make a Shew of mathematical Reasoning, without advancing one Step in real Knowledge.

SECT. 5. *Coroll. 3.*

I apprehend the Account that hath been given of the Nature of proper and improper Quantity may also throw some Light upon the Controversy about the Force of moving Bodies, which long exercised the Pens of many Mathematicians, and for what I know is rather drop'd than ended; to the no small Scandal of Mathematics, which hath always boasted of a Degree of Evidence, inconsistent with Debates that can be brought to no Issue.

Tho' Philosophers on both Sides agree with one another, and with the Vulgar in this, That the Force of a moving Body is the same, while its Velocity is the same, is increased when its Velocity is increased, and diminished when that is diminished. But this vague Notion of Force, in which both Sides agree, tho' perhaps sufficient for common Discourse, yet is not sufficient to make it a Subject of mathematical Reasoning: In order to that, it must be more accurately defined, and so defined as to give

us a Measure of it, that we may understand what is meant by a double or a triple Force. The *Ratio* of one Force to another cannot be perceived but by a Measure; and that Measure must be settled not by mathematical Reasoning, but by a Definition. Let any one consider Force without relation to any other Quantity, and see whether he can conceive one Force exactly double to another; I am sure I cannot, nor shall, till I shall be endowed with some new Faculty; for I know nothing of Force but by its Effects, and therefore can measure it only by its Effects. Till Force then is defined, and by that Definition a Measure of it assigned, we fight in the dark about a vague Idea, which is not sufficiently determined to be admitted into any mathematical Proposition. And when such a Definition is given, the Controversy will presently be ended.

#### SECT. 6.

#### *Of the Newtonian Measure of Force.*

You say, the Force of a Body in Motion is as its Velocity: Either you mean to lay this down as a Definition as *Newton* himself has done; or you mean to affirm it as a Proposition capable of Proof. If you mean to lay it down as a Definition, it is no more than if you should say, I call that a double Force which gives a double Velocity to the same Body, a triple Force which gives a triple Velocity, and so on in Proportion. This I intirely agree to; no mathematical Definition of Force can be given that is more clear and simple, none that is more agreeable to the common Use of the Word in Language.

For,

For since all Men agree, that the Force of the Body being the same, the Velocity must also be the same; the Force being increased or diminished, the Velocity must be so also, what can be more natural or proper than to take the Velocity for the Measure of the Force?

Several other things might be advanced to shew that this Definition agrees best with the common popular Notion of the Word Force. If two Bodies meet directly with a Shock, which mutually destroys their Motion without producing any other sensible Effect, the Vulgar would pronounce, without Hesitation, that they met with equal Force; and so they do, according to the Measure of Force above laid down: For we find by Experience, that in this Case their Velocities are reciprocally as their Quantities of Matter. In Mechanics, where by a Machine two Powers or Weights are kept in *equilibrio*, the Vulgar would reckon that these Powers act with equal Force, and so by this Definition they do. The Power of Gravity being constant and uniform, any one would expect that it should give equal Degrees of Force to a Body in equal Times, and so by this Definition it does. So that this Definition is not only clear and simple, but it agrees best with the Use of the Word Force in common Language, and this I think is all that can be desired in a Definition.

But if you are not satisfied with laying it down as a Definition, that the Force of a Body is as its Velocity, but will needs prove it by Demonstration or Experiment; I must beg of you, before you take one Step in the Proof, to let me know what you

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mean by Force, and what by a double or a triple Force. This you must do by a Definition which contains a Measure of Force. Some primary Measure of Force must be taken for granted, or laid down by way of Definition; otherwise we can never reason about its Quantity. And why then may you not take the Velocity for the primary Measure as well as any other? You will find none that is more simple, more distinct, or more agreeable to the common Use of the Word Force: And he that rejects one Definition that has these Properties, has equal Right to reject any other. I say then, that it is impossible, by mathematical Reasoning or Experiment, to prove that the Force of a Body is as its Velocity, without taking for granted the thing you would prove, or something else that is no more evident than the thing to be proved.

#### SECT. 7.

#### *Of the Leibnitzian Measure of Force.*

Let us next hear the *Leibnitzian*, who says, that the Force of a Body is as the Square of its Velocity. If he lays this down as a Definition, I shall rather agree to it, than quarrel about Words, and for the future shall understand him, by a quadruple Force to mean that which gives a double Velocity, by 9 times the Force that which gives three times the Velocity, and so on in duplicate Proportion. While he keeps by his Definition, it will not necessarily lead him into any Error in Mathematics or Mechanics. For, however paradoxical his Conclusions may appear, however different in Words from theirs  
who

who measure Force by the simple *Ratio* of the Velocity; they will in their Meaning be the same: Just as he who would call a Foot twenty-four Inches, without changing other Measures of Length, when he says a Yard contains a Foot and a half, means the very same as you do, when you say a Yard contains three Feet.

But tho' I allow this Measure of Force to be distinct, and cannot charge it with Falshood, for no Definition can be false, yet I say in the first place, it is less simple than the other; for why should a duplicate *Ratio* be used where the simple *Ratio* will do as well? In the next place, this Measure of Force is less agreeable to the common Use of the Word Force, as hath been shewn above; and this indeed is all that the many laboured Arguments and Experiments, brought to overturn it, do prove. This also is evident, from the Paradoxes into which it has led its Defenders.

We are next to consider the Pretences of the *Leibnitzian*, who will undertake to prove by Demonstration, or Experiment, that Force is as the Square of the Velocity. I ask him first, what he lays down for the first Measure of Force? The only Measure I remember to have been given by the Philosophers of that Side, and which seems first of all to have led *Leibnitz* into his Notion of Force, is this: The Height to which a Body is impell'd by any impressed Force, is, says he, the whole Effect of that Force, and therefore must be proportional to the Cause: But this Height is found to be as the Square of the Velocity which the Body had at the Beginning of its Motion.

In this Argument I apprehend that great Man has been extremely unfortunate. For, *1<sup>st</sup>*, Whereas all Proof should be taken from Principles that are common to both Sides, in order to prove a thing we deny, he assumes a Principle which we think farther from the Truth; namely, that the Height to which the Body rises is the whole Effect of the Impulse, and ought to be the whole Measure of it. *2<sup>dly</sup>*, His Reasoning serves as well against him as for him: For may I not plead with as good Reason at least thus? The Velocity given by an impressed Force is the whole Effect of that impressed Force; and therefore the Force must be as the Velocity. *3<sup>dly</sup>*, Supposing the Height to which the Body is raised to be the Measure of the Force, this Principle overturns the Conclusion he would establish by it, as well as that which he opposes. For, supposing the first Velocity of the Body to be still the same; the Height to which it rises will be increased, if the Power of Gravity is diminished; and diminished, if the Power of Gravity is increased. Bodies descend slower at the Equator, and faster towards the Poles, as is found by Experiments made on Pendulums. If then a Body is driven upwards at the Equator with a given Velocity, and the same Body is afterwards driven upwards at *Leipsick* with the same Velocity, the Height to which it rises in the former Case will be greater than in the latter; and therefore, according to his Reasoning, its Force was greater in the former Case; but the Velocity in both was the same; consequently the Force is not as the Square of the Velocity any more than as the Velocity.

*Reflections on this Controversy.*

Upon the whole, I cannot but think the Controversists on both Sides have had a very hard Task; the one to prove, by mathematical Reasoning and Experiment, what ought to be taken for granted, the other by the same means to prove what might be granted, making some Allowance for Impropriety of Expression, but can never be proved.

If some Mathematician should take it in his Head to affirm, that the Velocity of a Body is not as the Space it passes over in a given Time, but as the Square of that Space; you might bring mathematical Arguments and Experiments to confute him; but you would never by these force him to yield, if he was ingenuous in his Way; because you have no common Principles left you to argue from, and you differ from one another, not in a mathematical Proposition, but in a mathematical Definition.

Suppose a Philosopher has consider'd only that Measure of centripetal Force which is proportional to the Velocity generated by it in a given Time, and from this Measure deduces several Propositions. Another Philosopher in a distant Country, who has the same general Notion of centripetal Force, takes the Velocity generated by it, and the Quantity of Matter together, as the Measure of it. From this he deduces several Conclusions, that seem directly contrary to those of the other. Thereupon a serious Controversy is begun, whether centripetal Force be as the Velocity, or as the Velocity and Quantity of Matter taken together. Much mathematical and  
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experimental Dust is raised; and yet neither Party can ever be brought to yield; for they are both in the right, only they have been unlucky in giving the same Name to different mathematical Conceptions. Had they distinguished these Measures of centripetal Force as *Newton* has done, calling the one *Vis centripetæ Quantitatis acceleratrix*, the other *Quantitas motrix*; all Appearance of Contradiction had ceased, and their Propositions, which seem so contrary, had exactly tallied.

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VI. *A Letter from Rich. Hassel Esq; F.R.S. to Peter Daval Esq; Secr. R. S. concerning a large Piece of a Lath being thrust into a Man's Eye, who recover'd of it.*

S I R,

*Read Nov. 10. 1748.* I THOUGHT the following Case so extraordinary as to be worth the Notice of the *Royal Society*. If you think so too, I beg you to communicate it.

On Sunday the 17<sup>th</sup> of Jan. 1747. *Henry Halscy*, of *South Mims*, Labourer, thrust a long Lath with great Violence into the great *Canthus* of the left Eye of *Edward Roberts* of the same Place Labourer, which broke off quite short; so that a Piece two Inches and near a half long, half an Inch wide, and above a quarter of an Inch thick, (*see TAB. I.*) remained in his Head, and was so deeply buried there, that it could scarce  
be